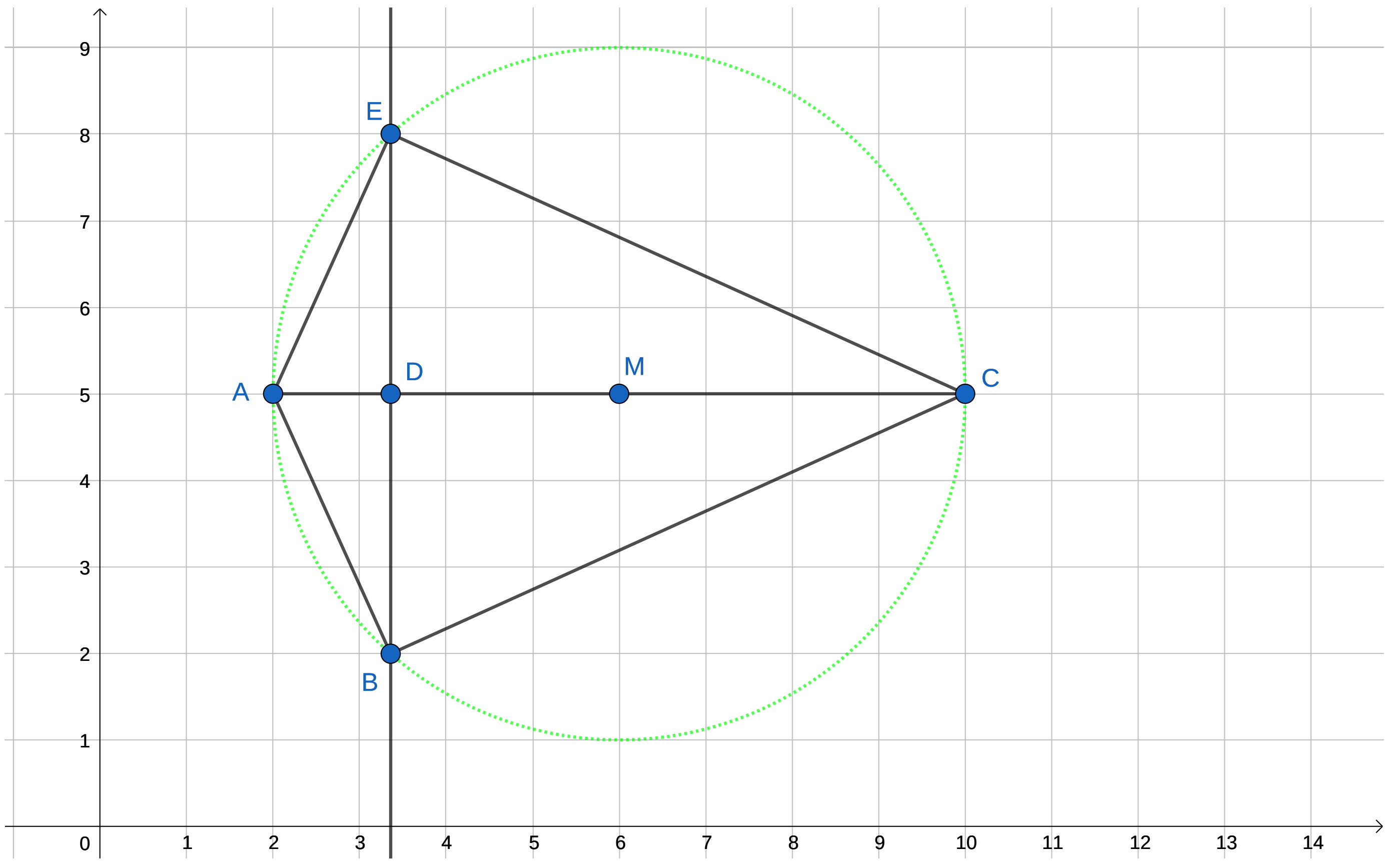
Demo Problem 3

1 Problem

ABC is a right triangle at vertex B. Point M is the midpoint of [A,C]. Line (B,D) is perpendicular on line (A,C). Point E is symmetric to point B with respect to point D. Prove that quadrilateral ABCE is cyclic.

2 Diagram



3 Input to the Program

3.1 Premises

Euclidean Description of the Problem
Triangle ABC is right at B.
M is the midpoint of [A,C].
(B,D) is perpendicular on line (A,C).
E is symmetric to B with respect to D.
Line (A,D,M,C).

Cartesian Description of the Diagram
Point A has coordinates (2,5).
Point B has coordinates (3.36,2).
Point C has coordinates (10,5).
Point D has coordinates (3.36,5).
Point E has coordinates (3.36,8).
Point M has coordinates (6,5).

3.2 Goal

Prove that quadrilateral ABCE is cyclic.

4 Notes

- In this particular example, the line (A,D,M,C) is specified because in the general case, there could be more than one way of drawing the diagram that could result in different positioning of points within a line.
- The circle is not part of the problem, it is drawn for illustrative purposes only.

5 Proof

Start

The quadrilateral ABCE will be proved cyclic quadrilateral by showing that its opposite angles are supplementary

1. The opposite angles [A,B,C] and [A,E,C] will be proved supplementary

Start

Angles [A,B,C] and [A,E,C] will be proved supplementary by showing they are both right

1. We prove that angle [A,B,C] is right

Start

The angle [A,B,C] will be proved right by proving it is the right angle of a right-angled triangle

Start

The angle is right because it is the right angle of the right triangle ABC

End

Therefore, angle [A,B,C] is a right angle

End

2. We prove that angle [A,E,C] is right

Start

The angle [A,E,C] will be proved right by showing it is composed of complementary angles [C,E,D] and [A,E,D]

Start

Angles [C,E,D] and [A,E,D] will be proved complementary by showing they are equal to angles that are complementary

1. We prove that angles [C,E,D] and [C,B,D] are equal

Start

Angle equality of [C,E,D] and [C,B,D] will be proved using SAS triangle congruence

The two triangles CDE and BCD have respectively:

1. We prove that segments [C,D] and [C,D] are equal

Start

The two segments are identical

End

2. We prove that segments [D,E] and [B,D] are equal

Start

Point E is symmetric to point B with respect to point D

End

3. We prove that angles [C,D,E] and [B,D,C] are equal

Start

Angles [B,D,C] and [C,D,E] will be proved both right and hence equal

1. We prove that angle [B,D,C] is right

Start

The angle [B,D,C] will be proved right by proving it is the same as another right angle [b,d,m]

Start

Angle [B,D,M] right because it is the same as angle [B,D,C] which is right from the premises

End

Therefore, angle [B,D,C] is a right angle

End

2. We prove that angle [C,D,E] is right

Start

The angle [C,D,E] will be proved right by proving it is equal to angle [A,D,B] which is to be proved right
1. We prove that angle [A,D,B] is right
Start
The angle [A,D,B] will be proved right by proving it is equal to angle [B,D,C] which is to be proved right
1. We prove that angle [B,D,C] is right
Start
Angle is formed by the intersection of two perpendicular lines
End
2. We prove that angles [A,D,B] and [B,D,C] are equal
Start
Angles [A,D,B] and [B,D,C] will be proved both right and hence equal
1. We prove that angle [A,D,B] is right
Start
Angle is formed by the intersection of two perpendicular lines
End
2. We prove that angle [B,D,C] is right
Start
Angle is formed by the intersection of two perpendicular lines
End
Therefore, angles [A,D,B] and [B,D,C] are equal
End
Therefore, angle [A,D,B] is a right angle
End
2. We prove that angles [C,D,E] and [A,D,B] are equal
Start
Angles [A,D,B] and [C,D,E] are equal because they are vertically-opposite angles
End
Therefore, angle [C,D,E] is a right angle
End
Therefore, angles [B,D,C] and [C,D,E] are equal
End
Therefore:
Triangles CDE and BCD are congruent, and hence: angles [C,E,D] and [C,B,D] are equal
End
2. We prove that angles [A,E,D] and [A,B,D] are equal
Start
Angle equality of [A,E,D] and [A,B,D] will be proved using SAS triangle congruence
The two triangles ADE and ABD have respectively:
1. We prove that segments [A,D] and [A,D] are equal
Start
The two segments are identical
End
2. We prove that segments [D,E] and [B,D] are equal
Start
Point E is symmetric to point B with respect to point D

End
3. We prove that angles [A,D,E] and [A,D,B] are equal
Start
Angles [A,D,B] and [A,D,E] will be proved both right and hence equal
1. We prove that angle [A,D,B] is right
Start
The angle [A,D,B] will be proved right by proving it is equal to angle [B,D,C] which is to be proved right
1. We prove that angle [B,D,C] is right
Start
Angle is formed by the intersection of two perpendicular lines
End
2. We prove that angles [A,D,B] and [B,D,C] are equal
Start
Angles [A,D,B] and [B,D,C] will be proved both right and hence equal
1. We prove that angle [A,D,B] is right
Start
Angle is formed by the intersection of two perpendicular lines
End
2. We prove that angle [B,D,C] is right
Start
Angle is formed by the intersection of two perpendicular lines
End
Therefore, angles [A,D,B] and [B,D,C] are equal
End
Therefore, angle [A,D,B] is a right angle
End
2. We prove that angle [A,D,E] is right
Start
The angle [A,D,E] will be proved right by proving it is equal to angle [B,D,C] which is to be proved right
1. We prove that angle [B,D,C] is right
Start
The angle [B,D,C] will be proved right by proving it is the same as another right angle [b,d,m]
Start
Angle [B,D,M] right because it is the same as angle [B,D,C] which is right from the premises
End
Therefore, angle [B,D,C] is a right angle
End
2. We prove that angles [A,D,E] and [B,D,C] are equal
Start
Angles [B,D,C] and [A,D,E] are equal because they are vertically-opposite angles
End
Therefore, angle [A,D,E] is a right angle
End
Therefore, angles [A,D,B] and [A,D,E] are equal
End

Therefore:

Triangles ADE and ABD are congruent, and hence: angles [A,E,D] and [A,B,D] are equal

End

3. We prove that angles [C,B,D] and [A,B,D] are complementary

Start

Angles [C,B,D] and [A,B,D] are complementary because form a right angle [a,b,c] which we prove below

Start

The angle [A,B,C] will be proved right by proving it is the right angle of a right-angled triangle

Start

The angle is right because it is the right angle of the right triangle ABC

End

Therefore, angle [A,B,C] is a right angle

End

Therefore: angles [C,B,D] and [A,B,D] are complementary

End

Therefore: angles [C,E,D] and [A,E,D] are complementary

End

Therefore, angle [A,E,C] is a right angle

End

Therefore: angles [A,B,C] and [A,E,C] are supplementary

End

2. Since the sum of angles in a quadrilateral is 360 degrees, and knowing that [A,B,C] and [A,E,C] are supplementary,

We conclude that the sum of the remaining angles [B,A,E] and [B,C,E] is 180 degrees i.e. they are supplementary as well

Therefore, quadrilateral ABCE is a cyclic quadrilateral

End